On the Possibility of Development of the Explosion Instability in a Two-Component Gravitating System

A. S. Kingsep, Yu. A. Fridman

We obtain an expression for the energy of the density wave propagating in a multicomponent gravitating medium in the form well known from electrodynamics. Using the above, the possibility of "triple production" of the quasiparticles, or waves, with their energies summing up to zero, in a nonequilibrium medium is demonstrated. That kind of resonance wave interaction is shown to result in the development of an explosion instability. By the method developed in plasma physics, the characteristic time of the instability is evaluated.

1 Introduction

One would judge dark matter to be a natural object for the collisionless hydrodynamics of multicomponent gravitating systems, in which the particle movement is governed by the self-consistent gravitational potential. Up till now, its particles have been manifesting themselves solely in gravitational interaction, hence the lack of the evidence on the question of their nature (see for instance [1]). In spite of certain well-known achievements of the last years (like a bona fide Hubble photograph of 2004, of the two clasters colliding, which is strongly believed to have captured a ring of dark matter), the question of the very existence of the dark matter, raised by Fritz Zwicky in 1937 (see [2]), is still unaswered [3].

Development of the instabilities in a multicomponent system is determined by the hydrodynamical (or kinetic) parameters of the media composing it. Traces of structures or turbulences owing their appearance to this or that instability would be impossible to observe in the dark matter, but they should be present, say, in the gaseous disk. A list of instabilities capable of arising in the system "gaseous disk — dark matter" might come handy when one sets to drawing some conclusions concerning hydrodynamical (or kinetic) properties of the dark matter as a medium. Linear hydrodynamical beam-jeans instabilities of a multicomponent gravitating medium are described, for instance, in [4], [5]. If the media are sufficiently hot, the kinetic beam-jeans instability can develop instead of the hydrodynamical one. If the system is stable in the linear approximation (e. g., the large-scale instabilities are stabilized by rotation [6]), non-linear scenarios of the development of instabilities enter the game.

2 Energy of the density wave in a gravitating medium

In this section, our aim is to obtain an expression for the energy of the density wave in a gravitating medium in the form convenient for further investigations.

In [10], while deriving the formula expressing the energy of the density wave in an ordinary gas (i. e., self-gravitation amounting to effective zero), total energy of the unit volume of a homogeneous medium is given as follows: $E = \rho\epsilon + \frac{\rho v^2}{2}$, ρ being the density of the medium, ϵ — the mass density of the internal density of the medium, v — its total velocity. Taking self-gravitation into account, one should add a term corresponding to the potential energy $\frac{1}{2}\rho\psi$, where ψ is the gravitational potential. Apart from that following the standard procedure, for the density wave in the motionless medium we get:

$$E_{wave} = \frac{1}{2}\frac{c_s^2}{\rho_0}\tilde{\rho}^2 + \rho_0\frac{\tilde{v}^2}{2} + \frac{1}{2}\tilde{\rho}\tilde{\psi} \qquad (1)$$

Here the "tilde" symbol marks the perturbed values while the index "0" stands for the equilibrium, c_s is the speed of the sound in the gas (we consider the isotermic case here, so that $c_s^2 = p/\rho$, p being the gas pressure). In a frame of reference where the medium has the velocity v_0, in the expression for the density of the energy of the wave one more term would appear, amounting to $\tilde{\rho} v_0 \tilde{v}$.

In the more general case of multicomponent gravitating medium with the plane-parallel relative motion of the components, the energy of the density wave per unit volume acquires the form:

$$E_{wave} = \sum_i \left(\frac{\rho_{0i}\tilde{v}_i^2}{2} + \tilde{\rho}_i \mathbf{v_{0i}} \tilde{\mathbf{v}}_\mathbf{i} + \frac{c_{si}^2 \tilde{\rho}_i^2}{2\rho_{0i}} + \frac{1}{2}\tilde{\rho}_i\tilde{\psi}_i \right) \qquad (2)$$

where the index "i" denotes the (both perturbed and pertaining to the equilibrium state) parameters of the component of that same number.

The equations of gravihydrodynamics for the multicomponent gravitating medium have the following form:

$$\left\{ \begin{array}{rcl} \frac{\partial \rho_i}{\partial t} + div(\rho_i \mathbf{v_i}) & = & 0 \\ \frac{\partial \mathbf{v_i}}{\partial t} + (\mathbf{v_i}\nabla)\mathbf{v_i} & = & -\nabla\psi - \frac{\nabla p_i}{\rho_i} \\ \Delta\psi & = & 4\pi G \sum_i \rho_i \end{array} \right. \qquad (3)$$

"i" being any integer from 1 to n, n — the number of the components of the system under consideration.

Linearizing the equations and expanding them into Fourier series, for any given harmonic $\propto e^{-i\omega t + i\mathbf{kr}}$ we get

$$\tilde{\rho}_i = \frac{\rho_{0i} k^2 \tilde{\psi}}{(\omega - \mathbf{k v_{0i}})^2 - k^2 c_{si}^2} \qquad (4)$$

$$\tilde{\mathbf{v}}_\mathbf{i} = \frac{\mathbf{k}\tilde{\psi}\,(\omega - \mathbf{k v_{0i}})}{(\omega - \mathbf{k v_{0i}})^2 - k^2 c_{si}^2} \qquad (5)$$

Substituting the above into the expression (2) for the energy of the density wave in a multicomponent medium, we get:

$$E_{wave} = k^2\tilde{\psi}^2 \sum_i \frac{\rho_{0i}\omega\left(\omega - \mathbf{k}\mathbf{v_{0i}}\right)}{\left(\left(\omega - \mathbf{k}\mathbf{v_{0i}}\right)^2 - k^2c_{si}^2\right)^2} \tag{6}$$

This is almost what we have aimed at.

Now, the dispersion equation derived from the linearized equations of gravihydrodynamics:

$$\varepsilon_0 = 1 + \sum_i \frac{\omega_{ji}^2}{\left(\omega - \mathbf{k}\mathbf{v}_{0i}\right)^2 - k^2c_{si}^2} = 0, \tag{7}$$

ω_{ji} being the jeans frequency of the component number i of the medium, defined in the usual way by $\omega_{ji}^2 = 4\pi G\rho_{0i}$, G being the newtonian gravitational constant, and the physical interpretation of the value ε_0 will be discussed a few passages below. Direct calculations show that for the values ω and $\mathbf{k}$ satisfying the dispersion equation (7),

$$E_{wave} = \frac{\partial(\varepsilon_0\omega)}{\partial\omega}\left(-\frac{|\tilde{\mathbf{g}}|^2}{8\pi G}\right) \tag{8}$$

where $\tilde{\mathbf{g}} = -\nabla\tilde{\psi}$ is the perturbed gravitational field strength (for a given Fourier harmonic $\tilde{\mathbf{g}} = -ik\tilde{\psi}$).

The structure of the expression (8) is fully analogous to the well-known results from classical electrodynamics [**?**] for the energy of the electromagnetic wave or the wave of the electric charge density in a dielectric medium. The difference rests in the sign (and in the presence, in the denominator, of the newtonian constant G), reflecting the difference in the nature of coulombian and newtonian interaction).

The value ε_0, analogous to the dielectric permettivity of a medium to the electric charge density waves, characterizes the responce of the density distribution in the various components of the gravitating medium to the perturbation of the gravitational potential, as it can be seen, for instance, from the expression for the perturbed density (4).

So we see that the sign of the energy of a density wave in a multicomponent gravitating medium is determined by the sign of $\dfrac{\partial\varepsilon_0}{\partial\omega}$.

3 On the Possibility of Simultaneous Existence of the Waves of Positive and Negative Energy in a Multicomponent Medium

Consider the simplest case of a multicomponent medium — a case of a medium consisting of two streams of matter. Let both media composing the system have the same hydrodynamical parameters (the isothermic sound speed c_s and jeans frequency $\omega_j^2 = 4\pi G\rho$)

and move through each other with the velocities v_0 and $-v_0$. For "one-dimensional" wave ($\mathbf{k}\|\mathbf{v}_0$) [1] in such a medium, (7) takes the form:

$$\varepsilon_0 = 1 + \frac{\omega_j^2}{(\omega - kv_0)^2 - k^2 c_s^2} + \frac{\omega_j^2}{(\omega + kv_0)^2 - k^2 c_s^2} = 0 \qquad (9)$$

A sketch of behaviour of the left side of the dispersion equation (as dependent on frequency) is shown on Fig. 1.

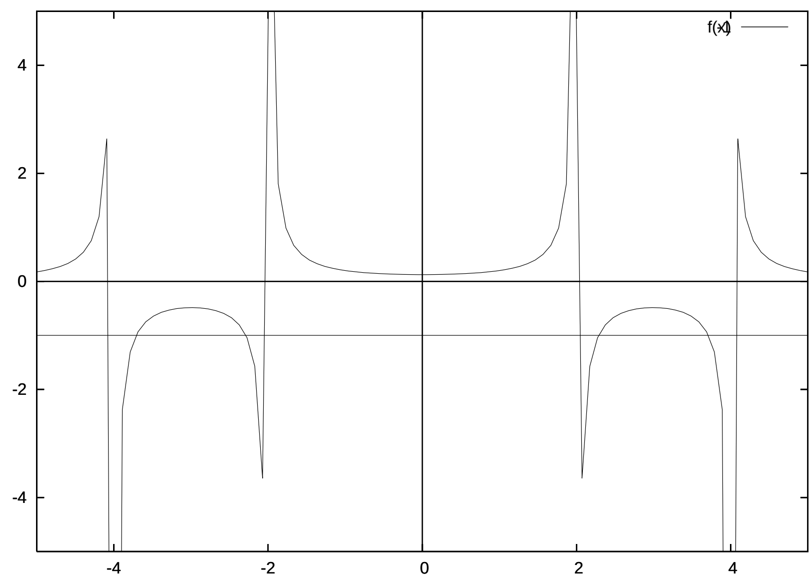

Figure 1: The left side of the dispersion equation against the wave frequency

If the curves lying in the lower semiplane intersect the straight line $y = -1$, corresponding to $\varepsilon_0 = 0$, the dispersion equation has four real solutions (i. e., such a two-streamed system is stable in the linear approximation), two of them corresponding to the positive, and two — to the negative energy of the wave. (All the above holds true in the "supersonic" case, when $v_0 > c_s$ — in the other case the sketch on Fig. 1 would have looked differently.)

It is easy to show that there exists a range of parameters in which such a system would be stable in the linear approximation. In this case, a wave propagating in the system can have either positive or negative sign in terms of energy. The dispersion equation (9) can be reduced to biquadratic. Introduce a nondismensional constant η in such a way that:

$$v_0^2 = (1+\eta)c_s^2$$

[1]The case of $\mathbf{k}\perp\mathbf{v}_0$, as can be easily shown, corresponds to an ordinary jeans instability. But the waves of large wavelength ($k^2 c_s^2 < 2\omega_j^2$) are excluded from out consideration anyway — by the limits of validity of our plane approximation as well as by the properties of real astronomical self-gravitating objects and their rotational geometry, in which the large-scale instabilities tend to be suppressed, see, for instance, [6]

The direct calculations show that the solutions ω^2 of the dispersion equation (9) are real when $\eta < 0$. If $\eta > 0$, the solutions ω^2 are real whenever

$$\left[\begin{array}{r} 0 \leq k^2c_s^2 \leq \frac{\omega_j^2\left(\sqrt{\eta+1}-\sqrt{\eta}\right)}{2\sqrt{\eta+1}} \\ k^2c_s^2 \geq \frac{\omega_j^2\left(\sqrt{\eta+1}+\sqrt{\eta}\right)}{2\sqrt{\eta+1}} \end{array}\right.$$

and amount to

$$(\omega^2)_{1,2} = k^2(2+\eta)c_s^2 - \omega_j^2 \pm \sqrt{4(\eta+1)k^4c_s^4 - 4(\eta+1)k^2c_s^2\omega_j^2 + \omega_j^4} \qquad (10)$$

Note that in the particular case $k^2c_s^2 = \omega_j^2$ the solutions of the dispersion equation take the form:

$$\begin{array}{rcl} (\omega^2)_1 & = & (2+\eta)\omega_j^2 \\ (\omega^2)_2 & = & \eta\omega_j^2 \end{array}$$

We see that, in this case, if $\eta > 0$, the dispersion equation has four real solutions, two of them corresponding to the positive values of the wave energy and two — to the negative ones.

Being interested in the solutions that are stable in the linear approximation (and thus corresponding to the ordinary (longtitudinal) density waves), we should now find k for which ω^2 is nonnegative. If $\eta > 0$, then for k satisfying the condition

$$k^2c_s^2 \geq \omega_j^2\frac{\sqrt{\eta+1}+\sqrt{\eta}}{2\sqrt{\eta+1}} \qquad (11)$$

all the four solutions ω are real, two of them corresponding to the waves of the positive energy, and two — to the negative energy waves.

Checking the case $\eta = 0$ ($v_0 = c_s$), we see that the system is stable in the linear approximation if $k^2c_s^2 \geq \omega_j^2$. If $\eta < 0$, the system demonstrates linear stabilities towards the perturbations for which $k^2c_s^2 \geq \frac{2\omega_j}{|\eta|}$. (One should remember that, when $\eta < 0$, $|\eta| \leq 1$ by definition.)

4 The Resonance Three, or Non-linear Interaction Resulting in Explosion Instability

Suppose that, being interested into all sorts of possible perturbations of our two-streamed medium, we stick to the range of linear stability.

We now consider plane waves as quasi-particles. All further calculations will be carried out in the random phase approximation: we consider the waves (interpreted as quasiparticles) to be incoherent. The quantities necessary for evaluation of the probability of the event (resonance effect or any collision) are provided by the second correction to the gravihydrodynamical equations; the dispersion equation characterizing the dependence of

the frequency upon the wave vector we have already obtained from the first correction, or linear approximation.

Quasi-particle collisions proceed with the conservation of energy and momentum. The important question is, can those conditions be held for a process sketched down on the scheme (Fig. 2) — triple quasi-particle production?

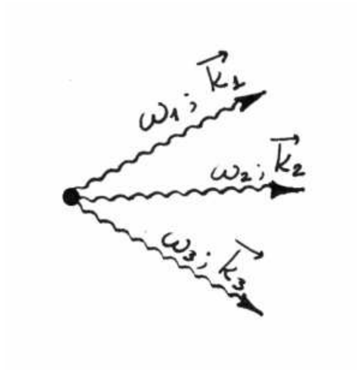

Fig. 2: Triple Quasi-particle Production

The conservation laws require that

$$\begin{aligned} \omega_1 + \omega_2 + \omega_3 &= 0 \\ \vec{k}_1 + \vec{k}_2 + \vec{k}_2 &= 0 \end{aligned} \tag{12}$$

As we shall see it below, such processes are indeed possible, and can be found even in one-dimensional model. Turn again to the waves with $\mathbf{k} \| \mathbf{v}_0$. As it can be seen from the plot on Fig. 1, for each k, to the negative energy wave corresponds the pair of ω having lesser absolute value. Note by ω^+ the solution of the dispersion equation corresponding to the positive energy of the wave, and by ω^- — to the negative. We are interested in the "resonance three", meaning simultaneous production of, say, one "positive" wave and two "negative" ones.

$$\begin{aligned} \left(\omega_1^-\right)^2 &= k_1^2 c_s^2 (2+\eta) - \omega_j^2 - \sqrt{4(\eta+1) k_1^4 c_s^4 - 4(\eta+1) k_1^2 c_s^2 \omega_j^2 + \omega_j^4} \\ \left(\omega_2^-\right)^2 &= k_2^2 c_s^2 (2+\eta) - \omega_j^2 - \sqrt{4(\eta+1) k_2^4 c_s^4 - 4(\eta+1) k_2^2 c_s^2 \omega_j^2 + \omega_j^4} \\ \left(\omega_2^+\right)^2 &= (k_1 - k_2)^2 c_s^2 (2+\eta) - \omega_j^2 + \\ &+ \sqrt{4(\eta+1)(k_1-k_2)^4 c_s^4 - 4(\eta+1)(k_1-k_2)^2 c_s^2 \omega_j^2 + \omega_j^4} \end{aligned} \tag{13}$$

The first negative wave can be represented by $\omega_1^-, -k_1$, the second wave of negative energy — by ω_2^-, k_2, and the wave of the positive energy, to complete our "resonance three", will be represented by $\omega^+ = -\omega_1^- - \omega_2^-$ and the corresponding wave number $k_1 - k_2$. (The dispersion law that we have here is symmetric function with respect to ω (when k is fixed) as well as with respect to k with fixed ω.) In order to prove that such a "resonance three" indeed does exist, consider the case $k^2 c_s^2 >> \omega_j^2$. In the second approximation with

respect to $\frac{\omega_j}{kc_s}$ we get:

$$
\begin{array}{rcl}
k_1 = -\sqrt{\eta+1}k & , & \omega_1 = \sqrt{\eta+1}\left(\sqrt{\eta+1}-1\right)kc_s - \frac{\omega_j^2}{2kc_s\sqrt{\eta+1}} \\
k_2 = k & , & \omega_2 = \left(\sqrt{\eta+1}-1\right)kc_s - \frac{\omega_j^2}{2kc_s\sqrt{\eta+1}} \\
k_3 = \left(\sqrt{\eta+1}-1\right)k & , & \omega_3 = -\eta kc_s + \frac{\omega_j^2}{2kc_s}\frac{\sqrt{\eta+1}+1}{\eta\sqrt{\eta+1}}
\end{array}
\tag{14}
$$

Solving the equation $\omega_1+\omega_2+\omega_3=0$ in terms of η gives the value $\eta=\frac{5}{4}$. For the short waves in the second approximation with respect to a small parameter we have presented "in quantities" the resonance three. The relative velocity of the two streams for the case under discussion must be $2v_0=3c_s$. Thus the triple quasi-particle production, which can take place in a two-streamed gravitating medium, is described by the conservation laws (12).

Note by N_k the number of quasi-particles carrying the 4-momentum[2] k (corresponding to the perturbations with the wave vector $\mathbf{k}$ and the frequency $\omega_{\mathbf{k}}$, $\mathbf{k}$ and $\omega_{\mathbf{k}}$ being related to each other by the dispersion equation). Supposing $N_k >> 1$ one can write:

$$
\begin{aligned}
\frac{\partial N_{k_1}}{\partial t} &= \sum_{k_2} Q(k_1,k_2)\left[(N_{k_1}+1)(N_{k_2}+1)(N_{-k_1-k_2}+1) - N_{k_1}N_{k_2}N_{-k_1-k_2}\right] \approx \\
&\approx \sum_{k_2} Q(k_1,k_2)\left[N_{k_1}N_{k_2} + N_{k_2}N_{-k_1-k_2} + N_{k_1}N_{-k_1-k_2}\right]
\end{aligned}
\tag{15}
$$

Here $Q(k_1,k_2)$ is the possibility of the triple quasi-particle production (or annihilation) of the quasi-particles with the momenta $(k_1, k_2, -k_1-k_2)$. As it is noted in [9], after the act of triple quasi-particle production is repeated quite many times, the number of quasi-particles of the three sorts that participate in the act can be considered roughly the same. Hence dismissing the indexes, we get:

$$
\frac{dN}{dt} = wN^2
$$

which leads us directly to the formula of the explosion instability:

$$
N(t) = \frac{N_0}{1-N_0wt} \tag{16}
$$

(Here N is the number of quasi-particles of any of the three resonant sorts dependent on time, N_0 — the initial number of the quasi-particles. The coefficient w is to be interpreted as the possibility of the process, the value, inversely proportional to the time of development of the instability. In the next section we shall calculate it for the one-dimensional case.)

[2]Here we have put $\hbar = 1$.

5 Evaluation of the Time of Development of the Explosion Instability in a Two-Streamed Gravitating Medium

The time $t_0 = (N_0 w)^{-1}$ of development of the explosion instability (16) is inversely proportional to the value w — the possibility of the resonance process responsible for this instability. (Naturally, as one can see from (16), w is the possibility per N, the number of quasi-particles, which is itself a dimensional quantity.)

We shall use the method developed by certain authors for the decay instability in the random phases approximation in the beginning of 1960-ies. (See, for instance, [9].) Since the gaseous galactic disk, with its particles moving through the hypothetic halo of the dark matter, is in the turbulent state, the random phase approximation here seems adequate.

The linear dispersion equation was obtained in the first approximation with respect to the amplitude; considering the case of weak nonlinearity, one should turn to the second approximation.

Consider the Fourier component of the Poisson equation correcponding to the momentum $\mathbf{k}$ and frequency ω. For the sake of convenience introduce the 4-dimensional wave vector $k = (\omega, \mathbf{k})$. Whenever the inverse is not clearly stated, speaking of the 4-vector k we would always have in mind $k = (\omega_{\mathbf{k}}, \mathbf{k})$, where $\omega_{\mathbf{k}} = \omega(\mathbf{k})$ is the solution of the dispersion equation $\varepsilon_0(\omega, \mathbf{k}) = 0$. By the above relation a manifold in the 4-dimensional space is defined; the 4-dimensional integration appearing in the further calculations is performed over this manifold. Below, among the indexes of the Fourier harmonics, 4-vector k as well as the normal 3-vector $\mathbf{k}$, will be used.

$$- \mathbf{k}^2 \varepsilon_0(\omega, \mathbf{k}) \tilde{\psi}_k = 4\pi G (\rho_{1k}^{(2)} + \rho_{2k}^{(2)}) \tag{17}$$

Here, as well as in the previous sections, ε_0 is the gravitational analogue of the dielectric permettivity. Index (2) over the symbol of density ρ indicates the correction of the second approximation. In the linear case $\varepsilon_0(\omega, \mathbf{k}) = 0$ would have held; if we present the small nonlinear correction to the frequency as $\delta\omega_N + i\gamma_N$, then, as is easily seen, for our approximation

$$\varepsilon_0(\omega, \mathbf{k}) \approx \left. \frac{\partial \varepsilon_0}{\partial \omega} \right|_{\omega = \omega_{\mathbf{k}}} (\delta\omega_N + i\gamma_N)$$

Multiply both parts of the equality by some $\mathbf{k}'$-component of the perturbed gravitational potential $\psi_{\mathbf{k}'}$ and then average it over phases:

$$-(\delta\omega_N + i\gamma_N) \left. \frac{\partial \varepsilon_0}{\partial \omega} \right|_{\omega = \omega_{\mathbf{k}}} \mathbf{k}^2 I_k \delta(k + k') = 4\pi G \left\langle \left(\rho_{1k}^{(2)} + \rho_{2k}^{(2)} \right) \psi_{k'} \right\rangle$$

Here I_k is the spectral intensity ($< \psi_k \psi_{k'} >= I_k \delta(k + k')$ in the random phases approximation).

The energy of the plane density wave is

$$W_k = -\frac{1}{8\pi G}\left.\frac{\partial(\varepsilon_0\omega)}{\partial\omega}\right|_{\omega=\omega_{\mathbf{k}}} |g_k|^2$$

Since $g_k = -(\nabla\psi)_k = -i\mathbf{k}\psi_k$ and $\varepsilon_0(\omega_{\mathbf{k}}) = 0$, for the number of quasi-particle with the 4-momentum k one can write:

$$N_k = \frac{W_k}{\omega_{\mathbf{k}}} = -\frac{1}{8\pi G}\left.\frac{\partial\varepsilon_0}{\partial\omega}\right|_{\omega=\omega_{\mathbf{k}}} \mathbf{k}^2 I_k$$

In the expression for the nonlinear frequency shift $\delta\omega_N + i\gamma_N$ the real part can be neglected compared to $\omega_{\mathbf{k}}$ — it corresponds to the renormalization of the linear dispersion law. The imaginary part $i\gamma_N$ corresponds to the nonlinear spectre evolution. Note that for $\psi \sim e^{-i(\omega+i\gamma_N)}$ the number of quasi-particles will be $N_k \sim e^{2\gamma_N t}$, so that multiplification by γ_N for the function $N_k(t)$ is equivalent to the action of the operator of the differentiation over the time: $\gamma_N \sim \frac{1}{2}\frac{\partial}{\partial t}$. Taking all this into account, note that the equation (17) transforms into:

$$\frac{\partial N_k}{\partial t}\delta(k+k') = -i\left\langle \left(\rho^{(2)}_{1k} + \rho^{(2)}_{2k}\right)\psi_{k'}\right\rangle \tag{18}$$

On the other hand, the structure of the time evolution of the number of quasi-particles is determined by the equation (15), or, in the integral form,

$$\frac{\partial N_{\mathbf{k}}}{\partial t} = \int w(\mathbf{k},\mathbf{q})\delta(\omega_{\mathbf{k}} + \omega_{\mathbf{q}} + \omega_{-\mathbf{k}-\mathbf{q}})\left(N_{\mathbf{q}}N_{-\mathbf{k}-\mathbf{q}} + N_{\mathbf{k}}N_{\mathbf{q}} + N_{\mathbf{k}}N_{-\mathbf{k}-\mathbf{q}}\right)dq \tag{19}$$

To make the long calculations short, we now set to follow the evolution of only one term of the sum under the symbol of integration, say, $N_{\mathbf{q}}N_{-\mathbf{k}-\mathbf{q}}$. We shall sort of exstract it from the second correction to the density; the analogous expressions for the rest two terms will be easy to construct; besides, the coefficient (i. e. the possibility in which we are interested) preceding them in the expressions like 19 is the same. We take the second correction to the density in the following form:

$$\rho_{\alpha k} = \int \nu^{\alpha}_{\mathbf{k},\mathbf{q}}\left(\psi_q\psi_{-k-q} - \langle\psi_q\psi_{-k-q}\rangle\right)dq \tag{20}$$

Here $\alpha = 1,2$ is the number of the corresponding component of the medium, $\nu^{\alpha}_{\mathbf{k},\mathbf{q}}$ is the coefficient to be calculated a few passages later, from the Euler equations. The 3-dimensional indexes $\mathbf{k},\mathbf{q}$ are used instead of the 4-dimensional ones because three components of the wave vector are sufficient to fully define the coefficient ν. The average value $\langle\psi_q\psi_{-k-q}\rangle$ corresponds to the evolution of the phone and should be subtracted.

Substituting (20) into (18), we get:

$$\frac{\partial N_k}{\partial t}\delta(k+k') = -i\int\left(\sum_{\alpha=1,2}\nu^{\alpha}_{\mathbf{k},\mathbf{q}}\right)\langle\psi_{k'}\left(\psi_q\psi_{-k-q} - <\psi_q\psi_{-k-q}>\right)\rangle\, dq \tag{21}$$

Substitute now the Fourier-component $\psi_{k'}$ by its nonlinear expansion (through the density from (17)):

$$\psi^{(2)}_{k'} = -\frac{4\pi G}{k'^2\varepsilon_0(\omega', \mathbf{k}')} \int \left(\sum_{\alpha} \nu^{\alpha}_{\mathbf{k}',\mathbf{q}'}\right) (\psi_{q'}\psi_{-k'-q'} - < \psi_{q'}\psi_{-k'-q'} >)\, dq'$$

in the course of this substitution we would have to perform the averaging procedure. The randomness of the phases results in reducing of all the correlations to binary, so that:

$$\begin{aligned}\langle\psi_{q'}\psi_{-k'-q'}\psi_q\psi_{-k-q}\rangle =& < \psi_{q'}\psi_q >< \psi_{-k'-q'}\psi_{-k-q} > + < \psi_{q'}\psi_{-k'-q'} >< \psi_q\psi_{-k-q} > + \\ &+ < \psi_{q'}\psi_{-k-q} >< \psi_{-k'-q'}\psi_q >\end{aligned}$$

The middle term here vanishes by the virtue of (21), and the rest two terms are symmetrical with respect to replacement $q' \to -k' - q'$, so that one of them can be doubled and the other one written off:

$$\begin{aligned}\langle\psi_{q'}\psi_{-k'-q'}\psi_q\psi_{-k-q}\rangle &= 2I_qI_{-k-q}\delta(q+q')\delta(k+k') = \\ &= 2\cdot(8\pi G)^2 N_q N_{-k-q}\left[(\mathbf{k}+\mathbf{q})^2 \left.\frac{\partial\varepsilon_0}{\partial\omega}\right|_{\omega=\omega_{-\mathbf{k}-\mathbf{q}}} \mathbf{q}^2 \left.\frac{\partial\varepsilon_0}{\partial\omega}\right|_{\omega=\omega_{\mathbf{q}}}\right]^{-1} \times \\ &\times \delta(q+q')\delta(k+k')\end{aligned}$$

Using an approximate equality in terms of generalized functions:

$$\varepsilon_0^{-1}(\omega'_{\mathbf{k}'}, \mathbf{k}') \simeq \left[(\omega' - \omega'_{\mathbf{k}'}) \left.\frac{\partial\varepsilon_0}{\partial\omega}\right|_{\omega=\omega'_{\mathbf{k}'}}\right]^{-1} \simeq \left(\left.\frac{\partial\varepsilon_0}{\partial\omega}\right|_{\omega=\omega'_{\mathbf{k}'}}\right)^{-1} i\pi\delta(\omega' - \omega'_{\mathbf{k}'})$$

and substituting all that into (21), performing integration over dq' and dk', for the evolution of the number of quasi-particles (tracing the transformations of one of the first two terms of (19) — $N_{\mathbf{q}}N_{-\mathbf{k}-\mathbf{q}}$ — we end up with the following expression:

$$\begin{aligned}\frac{\partial N_k}{\partial t} &= i\tau \int dq (8\pi G)^3 (\sum \nu^{\alpha}_{\mathbf{k},\mathbf{q}})^2 \times \\ &\times \left[\mathbf{k}^2 \left.\frac{\partial\varepsilon_0}{\partial\omega}\right|_{\omega=\omega_{\mathbf{k}}} (\mathbf{k}+\mathbf{q})^2 \left.\frac{\partial\varepsilon_0}{\partial\omega}\right|_{\omega=\omega_{-\mathbf{k}-\mathbf{q}}} \mathbf{q}^2 \left.\frac{\partial\varepsilon_0}{\partial\omega}\right|_{\omega=\omega_{\mathbf{q}}}\right]^{-1} \times \\ &\times N_q N_{-k-q}\end{aligned} \tag{22}$$

Here τ is the dimensional coefficient, by the order of magnitude similar to L/v_0; $\tau = \frac{V}{(2\pi)^3}\int \delta(\omega' - \omega_{\mathbf{k}'})d\mathbf{k}'$. Thus we see that the expression for the probability has the following structure:

$$w = -(8\pi G)^3\pi\tau \left(\sum_{\alpha} \nu^{\alpha}_{\mathbf{k},\mathbf{q}}\right)^2 \left[\mathbf{k}^2 \left.\frac{\partial\varepsilon_0}{\partial\omega}\right|_{\omega=\omega_{\mathbf{k}}} (-\mathbf{k}-\mathbf{q})^2 \left.\frac{\partial\varepsilon_0}{\partial\omega}\right|_{\omega=\omega_{-\mathbf{k}-\mathbf{q}}} \mathbf{q}^2 \left.\frac{\partial\varepsilon_0}{\partial\omega}\right|_{\omega=\omega_{\mathbf{q}}}\right]^{-1} \tag{23}$$

Now one has to determine the coefficient $\nu^{\alpha}_{\mathbf{k},\mathbf{q}}$ given by relation (20). The second correction to the continuity equation is:

$$\frac{\partial \rho^{(2)}_{\alpha}}{\partial t} + \rho^{(1)}_{\alpha} div \mathbf{v}^{(1)}_{\alpha} + (\mathbf{v}^{(1)}_{\alpha}\nabla)\rho^{(1)}_{\alpha} + \rho_{0\alpha} div \mathbf{v}^{(2)}_{\alpha} + (\mathbf{v}_{0\alpha}\nabla)\rho^{(2)}_{\alpha} = 0 \tag{24}$$

The Euler equation in the second approximation:

$$\frac{\partial \mathbf{v}^{(2)}_{\alpha}}{\partial t} = -(\mathbf{v}^{(1)}_{\alpha}\nabla)\mathbf{v}^{(1)}_{\alpha} - (\mathbf{v}^{(0)}_{\alpha}\nabla)\mathbf{v}^{(2)}_{\alpha} \tag{25}$$

Fourier-expansion for $(\omega, \mathbf{k})$-component of the continuity equation gives:

$$\omega \rho^{(2)}_{\alpha,\mathbf{k}} = \int d\mathbf{q} \rho^{(1)}_{\alpha\mathbf{q}} \left((\mathbf{k}-\mathbf{q}), \mathbf{v}^{(1)}_{\alpha,\mathbf{k}-\mathbf{q}}\right) + \int d\mathbf{q} \left(\mathbf{q}, \mathbf{v}^{(1)}_{\alpha,\mathbf{k}-\mathbf{q}}\right) \rho^{(1)}_{\alpha,\mathbf{q}} + \rho_{0\alpha} \left(\mathbf{k}\mathbf{v}^{(2)}_{\alpha,\mathbf{k}}\right) + \rho^{(2)}_{\alpha,\mathbf{k}}(\mathbf{k}\mathbf{v}_{\mathbf{0}\alpha}) \tag{26}$$

Rewriting the same relation in more symmetrical form:

$$\rho^{(2)}_{\alpha,\mathbf{k}}(\omega - \mathbf{k}\mathbf{v}_{0\alpha}) = \frac{1}{2}\int d\mathbf{q} \left[\rho^{(1)}_{\alpha\mathbf{q}}(\mathbf{k}, \mathbf{v}^{(1)}_{\alpha,\mathbf{k}-\mathbf{q}}) + \rho^{(1)}_{\alpha,\mathbf{k}-\mathbf{q}}(\mathbf{k}, \mathbf{v}^{(1)}_{\alpha,\mathbf{q}})\right] + \rho_{0\alpha}(\mathbf{k}, \mathbf{v}^{(2)}_{\alpha,\mathbf{k}}) \tag{27}$$

The values of the linear approximation are related in the following way:

$$\rho^{(1)}_{\alpha,\mathbf{k}} = \rho_{\alpha 0}\frac{\mathbf{k}\mathbf{v}^{(1)}_{\alpha,\mathbf{k}}}{\omega - \mathbf{k}\mathbf{v}_{0\alpha}}$$

Fourier transform applied to the Euler equations gives:

$$(\omega - \mathbf{k}\mathbf{v}_{0\alpha})\mathbf{v}^{(2)}_{\alpha\mathbf{k}} = \frac{1}{2}\int d\mathbf{q} \left[\mathbf{v}^{(1)}_{\alpha\mathbf{q}}\left(\mathbf{q}, \mathbf{v}^{(1)}_{\alpha,\mathbf{k}-\mathbf{q}}\right) + \mathbf{v}^{(1)}_{\mathbf{k}-\mathbf{q}}\left(\mathbf{k}-\mathbf{q}, \mathbf{v}^{(1)}_{\mathbf{q}}\right)\right] \tag{28}$$

In order to obtain the result in the analitical form, we now turn to 1-D case. As in previous sections, let $\rho_{01} = \rho_{02} = \rho_0$; $v_{01} = -v_{02} = v_0$. Below the index ¡¡k¿¿, marking the Fourier component, will mean now the one-dimensional "wave number", not the 4-momentum.

From the Fourier transform of the linear approximation one easily obtains:

$$v^{(1)}_{1,k} = \frac{k(\omega_k - kv_0)}{(\omega_k - kv_0)^2 - k^2 c_s^2}\psi_k$$

$$\rho^{(1)}_{1,k} = \frac{k^2 \rho_0}{(\omega_k - kv_0)^2 - k^2 c_s^2}\psi_k$$

Changing $v_0 \to -v_0$ and (1) to (2) in the above expression, we get the similar relation for the second component.

Substituting (28) into (27) and taking into account the above expressions for the Fourier components of the velocity and density in the linear approximations as related to the corresponding Fourier component of the amplitude of the gravitational potential, also

having in mind that $\nu^1_{-k,q} = \nu^2_{k,q}$ due to the symmetry of the initial conditions, we obtain the coefficient:

$$\nu^2_{k,q} = \frac{1}{2}\rho_0 \times \frac{kq(k+q)}{A^-_q A^-_{-k-q}(\Omega^-_{-k})^2} \times T^{2,3}_1(-) \tag{29}$$

Here (in the above) for each $\chi = k, q, -k-q$ the following notations have been introduced:

$$\Omega^{\pm}_{\chi} = \omega_{\chi} \pm \chi v_0$$
$$A^{\pm}_{\chi} = (\Omega^{\pm}_{\chi})^2 - \chi^2 c_s^2$$

and

$$T^{2,3}_1(\pm) = q\Omega^{\pm}_{-k-q}\Omega^{\pm}_{-k} + (-k-q)\Omega^{\pm}_{q}\Omega^{\pm}_{-k} + (-k)\Omega^{\pm}_{q}\Omega^{\pm}_{-k-q}$$

For the first component of the medium (the stream moving with the positive velocity v_0) the corresponding coefficient $\nu^1_{k,q}$ can be obtained from $\nu^2_{k,q}$ by replacement $v_0 \to -v_0$. In the above passages, performing the Fourier transform, we have not usually mentioned the dimensional coefficient; it is easy to see that the latter will be important only in calculation of the coefficient τ, which in one-dimensional model is equal to:

$$\tau = \frac{L}{2\pi}\int \delta(\omega' - \omega_{k'})dk' = \frac{L}{2\pi}\left.\frac{dk'}{d\omega'}\right|_{\omega'=\omega_{k'}}$$

L being the characteristic scale of the problem, the linear size of the system in the direction parallel to the velocity of both streams. Calculating this expression for $k' = k$, $\omega_{k'} = \omega_k$, we get:

$$\tau = \frac{L}{2\pi} \times \frac{(A^+_k)^2\Omega_- + (A^-_k)^2\Omega_+}{(-\omega v_0 + k\eta c_s^2)(A^+_k)^2 + (\omega v_0 + k\eta c_s^2)(A^-_k)^2}$$

Thus the probability is, effectively, found. The expression for the probability even in one-dimensional case looks somewhat awkward:

$$w = \frac{\pi\tau}{4\rho_0} \times \frac{\left(T^{2,3}_1(-)A^+_q A^+_{-k-q}(\Omega^+_{-k})^2 + T^{2,3}_1(+)A^-_q A^-_{-k-q}(\Omega^-_{-k})^2\right)^2 (A^-_k)^2(A^+_k)^2}{(\Omega^+_k)^4(\Omega^-_k)^4 D_k D_q D_{-k-q}} \tag{30}$$

Here for all $\chi = k, q, -k-q$ the following notation is introduced:

$$D_{\chi} = \Omega^-_{\chi}(A^+_{\chi})^2 + \Omega^+_{\chi}(A^-_{\chi})^2$$

The expression obtained above, in principle, provides a possibility to evaluate the time of development of the instability, if the hydrodynamical parameters of the problem (i. e. density, speed of sound) are known and the "resonance three" are determined. In our case, since the two streams going through each other in the opposite directions are hydrodynamically identical, and the "resonance three" has been obtained, all we have to know to evaluate the time of development of the explosion instability is the density and the speed of sound.

For the resonance three determined in one of the previous sections, if the characteristic size L is taken to be the jeans size of one of the components, then the value w, calculated according to (30), acquires the following structure:

$$w = \frac{4\pi\omega_j^2}{\rho_0 c_s^2 \beta}$$

β being the relation of the jeans wavelength to the wavelength of the perturbation in question. Supposing that in gravihydrodynamics, like in plasma physics, the degree of the turbulence can be evaluated by the parameter $\frac{W}{nT} << 1$, where W is the density of the noise energy, we get the evaluation for the time of development of the explosion instability:

$$t_{ins} \sim \alpha(k) \frac{\rho_0 c_s^2}{n_0 T \omega_j}$$

$\alpha(k)$ being a dimensionless coefficient which is the less the greater the turbulence degree of the medium $\frac{W}{nT}$ is. If $W(k)$ has a pointed peak then for the value of k corresponding to that maximum $\alpha(k) << 1$. Since, obviously, the second factor in the expression for t_{ins} has the same order of magnitude as the inverse jeans frequency (say, $10^{15} - 10^{16}$ s for the interstellar gas), that would mean that, for those k, the instability would be able to develop. It is important to have in mind that what we deal with is a nonlinear process, achieving the infinite amplitude in finite times (ideally speaking). Comparing linear exponential instabilities we take the definitely fastest one in terms of increments; to say that one of the instabilities is fast enough to develop against the background of the other, we require a strong inequality $\gamma_1 >> \gamma_2$ (γ_1, γ_2 being the increments). But when one of the instabilities in question is explosion instability — the simple inequality is enough to conclude that it is faster[3].

Returning to the real astronomical situation, it should be noted that the matter of the gaseous disk and the dark matter can hardly be considered hydronamically identical. To take the hydrodynamical difference between the two streams into account should not change dramatically the expression for the probability: each coefficient $\nu_{k,q}^{\alpha}$, $\alpha = 1, 2$, will be expressed through the equilibrium density of the corresponding component, and the auxiliary constructions like $D_\chi^\pm$ would get additional indexes, so that:

$$A_\chi^+ = (\omega_\chi + \chi v_0)^2 - k^2 c_{s1}^2$$

$$A_\chi^- = (\omega_\chi - \chi v_0)^2 - k^2 c_{s2}^2$$

Yet the dispersion equation would not be analitically solvable any more; numerical calculations would be required.

[3] In our case the actual characteristic sizes are not, in fact, jeans wavelengths: they are determined by the conditions limiting the applicability of our plane approximation. In the real cosmic space geometry, however, the large-scale instabilities are stabilized by rotation.

6 Conclusions

In the present work, we have obtained the expression for the density wave in a gravitating medium, analogous to the corresponding formula in plasma physics. The analogy has been toyed with by the theoretists of both fields, but seems to have never been clearly formulated and demonstrated in the general case. The important role in this parallel between the two regions of physics is played by a physical value describing the responce of the distribution of the density of the gravitating matter to a change in the gravitational field. In electrostatics and electrodynamics the analogous value is the dielectric permettivity of the medium.

Having thus legitimized certain analogies between the plasma physics and physics of gravitating media that have been noted before, we concentrated on development of one of those analogies, related to the possibility of development of the explosion instability in a two-streamed gravitating medium.

We have shown that, just as in a two-streamed plasma, in the two-streamed gravitating medium simultaneous propagation of the waves of positive and negative energies is possible. On the example of two streams moving through each other in opposite directions and having the same hydrodynamic characteristics it was demonstrated that a multicomponent gravitating medium can provide the background for development of the explosion instability.

The explosion instability is faster than the linear, exponential instabilities; the amplitude of the perturbation reaches infinity within finite time. Using the standard method developed in plasma physics, we have demonstrated the way to calculate this time for one-dimensional model.

Stabilization of the explosion instability is achieved through restructuralization of the dispersion equation. The dependence of the frequency of the perturbation on the wave vector that we have used in our calculations was obtained from the linear approximation. For large enough amplitudes, this dependence changes its form, the "resonance three" do not resonate any more, so that triple production of these quasi-particles becomes impossible.

Classification of various scenarios, composition of a librarian's catalogue of all possible instabilities has been long recognised as an important task in plasma physics. In graviphysics, the instabilities responsible for the most part of the observed phenomena and structures in the Universe probably deserve no less attention on the part of physicists and astronomers.

The authors would like to express their gratitude to A. L. Barabanov, K. V. Chukbar and A. M. Fridman for fruitful discussions.

References

[1] A. V. Gurevich, K. P. Zybin, *Large-scale Structure of The Universe, The Analytic Theory*, Successes of Physical Sciences, vol. 165, issue No. 7, July 1995.

[2] F. Zwicky, *On the Masses of Nebulae and of Clusters of Nebulae*, Astrophysical Journal, vol. 86, p. 217, October 1937

[3] F. I. Copperstock, S. Tieu, *General Relativity Resolves Galactic Rotation Without Exotic Dark Matter*, lanl.arXiv.org, Astrophysics, July 2005

[4] V. L. Polyachenko, A. M. Fridman, *Oscillations and Instabilities in The Multi-Component Gravitating Medium*, JETP, vol. 81, issue No 1(7), 1981.

[5] A. M. Fridman, Yu. A. Fridman, *On The Instabilities of N-component Uniform Gravitating Medium*, AJ Letters, vol. 17, issue No. 2, 1991.

[6] V. L. Polyachenko, A. M. Fridman, *Equilibrium and Stability of The Gravitating Systems*, "Science", Moscow, 1976.

[7] A. B. Mikhailovski, V. I. Petviashvili, A. M. Fridman, *Explosion Instability of The Gravitating Disk*, JEPT Letters, vol. 26, p. 341 (1977)

[8] V. M. Dikasov, L. I. Rudakov, D. D. Ryutov, *On Interaction of The Waves with Negative Energy in A Slightly Turbulent Plasma*, JETP, vol. 48, issue No. 3, 1965.

[9] A. S. Kingsep, *Introduction to Nonlinear Plasma Physics*, M3 Press, Moscow, 2004.

[10] L. D. Landau, E. M. Lifshits, *Electrodynamics of Continuous Media*, Moscow, "Science", 1982.

[11] A. B. Mikhailovsky, *Theory of Plasma Instabilities*, vol. 1, Atomizdat, Moscow, 1975.

[12] A. S. Kingsep, *On the Role of the Nonlinear Effects in The Problem of The Anomalous Resistance of Plasma*, JETP, vol. 63, issue No. 2(8), 1972.

[13] A. B. Mikhailovski, A. M. Fridman, *The Beam Instability in Gravitating Media*, JETP, vol. 61, 1971.

[14] A. B. Mikhailovski, V. I. Petviashvili, A. M. Fridman *Nonlinear Theory of Stability of Rotating Gravitating Disk*, AJ, vol. 56. p. 279 (1979)

[15] B. B. Kadomtsev, A. B. Mikhailovski, A. V. Timofeev, *Waves with Negative Energy in Dispersing Media*, JETP, vol. 47, issue No. 6(12), 1964.

[16] L. E. Aranchuk, Yu. G. Kalinin, A. S. Kingsep, V. A. Skoryupin, V. V. Yankov, Thermalisation of The Directed Movement of Plasma Clusters by The Turbulent Skinned Discharge, JETP, vol. 71, 1976.

[17] A. S. Kingsep, Yu. V. Mokhov, K. V. Chukbar, *On The Nonlinear Phenomena Accompanying The Transportation of A Powerful Jet of Light Ions towards The Target*, "Plasma Physics", vol. 11, p. 282, 1985.

[18] I. E. Tamm, *The Basics of The Theory of Electricity*. Moscow, 1956.

[19] L. D. Landau, E. M. Lifshits, *The Field Theory*, "Science", Moscow, 1973.